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MIXED-MODEL BALANCING PROCEDURE (SPECIAL SOLUTION)

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عملية اتزان خط متعدد النماذج (حل خاص)

خلاصة - تعرف عملية توزيع المهام على جميع محطات أحد خطوط التجميع الخاصة بانتاج منتج ما مع أقل فقد في الوقت بمشكلة الاتزان . ويمكن أن ينتج نموذج أو عدة نماذج آتية للمنتج على خط تجميع واحد فالنوع الأول يعرف بالانتاج بخط التجميع الفردي النموذج ، أما النوع الثاني فيعرف بالانتاج بخط التجميع المتعدد النماذج ، وكلما ازدادت المنافسة في السوق يزداد الميل للانتاج بواسطة خط التجميع المتعدد . ينصب اهتمام هذا البحث على عملية اتزان النموذج المتعدد ، كما يطرح أسلوبا للاتزان يمكن استخدامه في عملية اتزان أى خط تجميع متعدد النماذج . وتتضح صلاحية هذا الأسلوب بقدر ما يقدمه من حلول مثلى للحالات الخاصة عند الانتاج بخط التجميع المتعدد النماذج . يناقش هذا البحث مثل هذه المشكلات ،كما يوضح خطوات الاجراء ويقدم برنامج كمبيوتر بلغة الفورتران لتناول اجراء الاتزان ، كما يستعرض حل مثال لحالة تجريبية ، ويكون ناتج البرنامج بحيث يستعرض أولا كل البيانات الأولية للمسألة ويليها حل الاتزان .

ABSTRACT- The problem of allocating tasks to work stations along an assembly line to produce a product with minimum idle time is known as the balancing problem. Assembly line production may be either one or more than one model of one product. The first case of production is known as the single-model assembly line production. The second case of production is known as the mixed-model assembly line production. As the market competition increases, the demand for mixed-model production increases.

The paper at hand is concerned with the mixed-model balancing problem and presents a balancing approach which can be applied to any mixed-model assembly line balancing problem.

This approach is suitable in so far as it can give optimal solutions* for special cases of mixed-model assembly line production.

Those problem cases are discussed in the present paper. The procedure steps are given and a FORTRAN Computer program is written to handle the balancing procedure. A test case example solution is displayed. The program output balancing solution gives all the problem data first and the resulting balancing solution second.

INTRODUCTION

In refering to assembly line balancing problem, the concern is to find a way of assigning tasks or jobs to a series of work stations along what is known as assembly line, so as to minimize the total idle time along the line. In single model assembly production there is only one model of one product assembled and produced along the line. The problem of single model balancing problem received much research effort. [(2), (3), (4), (5), (6), (7), (8), (11), (12), (13), (14), (15), (19), (20)].

In mixed model assembly production there are more than one model assembled along the line. In practice there are potentially more applications of mixed model assembly line production than single model. Examining the problem of mixed model balancing is different than that of single model. In case of mixed model balancing problem, the concern is to examine those situations where one assembly line has to deal with a variety of models (or products) with differing work content. This means that mixed model production involves not only the allocation of tasks to work stations along the assembly line (as the case of single model balancing) but may also has to consider the problem of work content variety between models produced along the line. There are four main approaches for the design of mixed-model assembly line production [Wester, L. and Kilbridge, M.D. (21), Thomopoulos, N.T. (16, 17, 18), Macaskill, J.L.C (9,10), and Abdel-Shafi, A.A. (1)]. The first three approaches dealt with the problem in two stages to research solution for the problem.

The first stage is a balancing procedure to allocate tasks to work stations, the second stage is a sequencing procedure to determine the models assembly priorities. The fourth approach dealt with the problem in one stage as a balancing problem only. The balancing procedure developed in the fourth approach uses the model weights in ranking tasks for assignments to work stations, i.e. the procedure gives priorities to the models having large quantities in product-mix. All the four approaches produce a balancing solution for the mixed-model production having a single station-task assignment. i.e. any particular task has to be performed (assigned) to only one work station for all the involved models.

The work presented in this paper is a new idea for balancing mixed-model production in general. The presented balancing procedure gives very good solutions in cases where models in production having nearly similar amounts of work content durations and at the same time each model work content performance is different from the others. The balancing approach presented here releasing the restriction of single station-task assignment used in the previous approaches (1,9,10,16,17,18,21). The presented approach allows any task to be assigned (performed) to more than one work station for the different involved models if the balance requires so. [The techniques developed before (1,9.10,16,17,18,21) allow any task to be assigned to only one work station for all the different involved models].

The balancing procedure in this work gives optimal solution where the following assembly line production conditions existed :

- Work content durations for each model (or product) is nearly the same
- Work content performance for each model (or product) is different from the others.
- Work training is not expensive and/or the assembly work requires little skillness.
- No much difference in model ratios between models in the production-mix.

Mixed - Model Balancing Procedure :-

The developed procedure is programed in FORTRAN IV language. The data feeded to the program is stored in three files. The first file contains all the task information, such as task durations, task-model involvements, and task precedence relationships. The second file contains all the models informations such as model ratios in the production schedule (number of units required from each model), and models specifications. The third file contains the balancing instructions, such as production rate or production cycle time, and control parameters for the program output to give different types of balancing results (full details output, Semi-details output, and Breif output). The approach solution procedure consists of three phases.

Phase I :

A preparation for the balance is made in this step to be ready for the use in next phase. A rank list is prepared for each individual model as follows :

1- For each individual model in turn, a positional weight for each involved task is calculated using the following formula :

$$(P_{ij})_w = t_i \; (I_j) + \sum_{L=1}^{n} t_L \; (F_i \; I_j)$$

2- For each model in turn, a model list of tasks is prepared in descending order of their positional weight $(P_{ij})_w$

Where :

$(P_{ij})_w$: Positional weight of task i for model j

$t_i (I_j)$: Task i duration when involved in model j, if not involved, then $t_i (I_j) = 0$

$t_L (F_i I_j)$: Taskduration of follower L, when both a follower of task i and involved in model j , if not, then $t_L (F_i I_j) = 0$

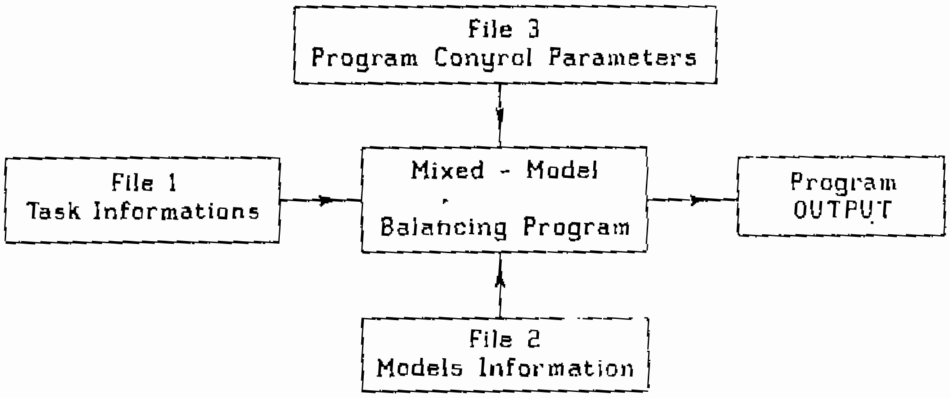
n : Number of tasks invoved in production for all models.

Phase II :

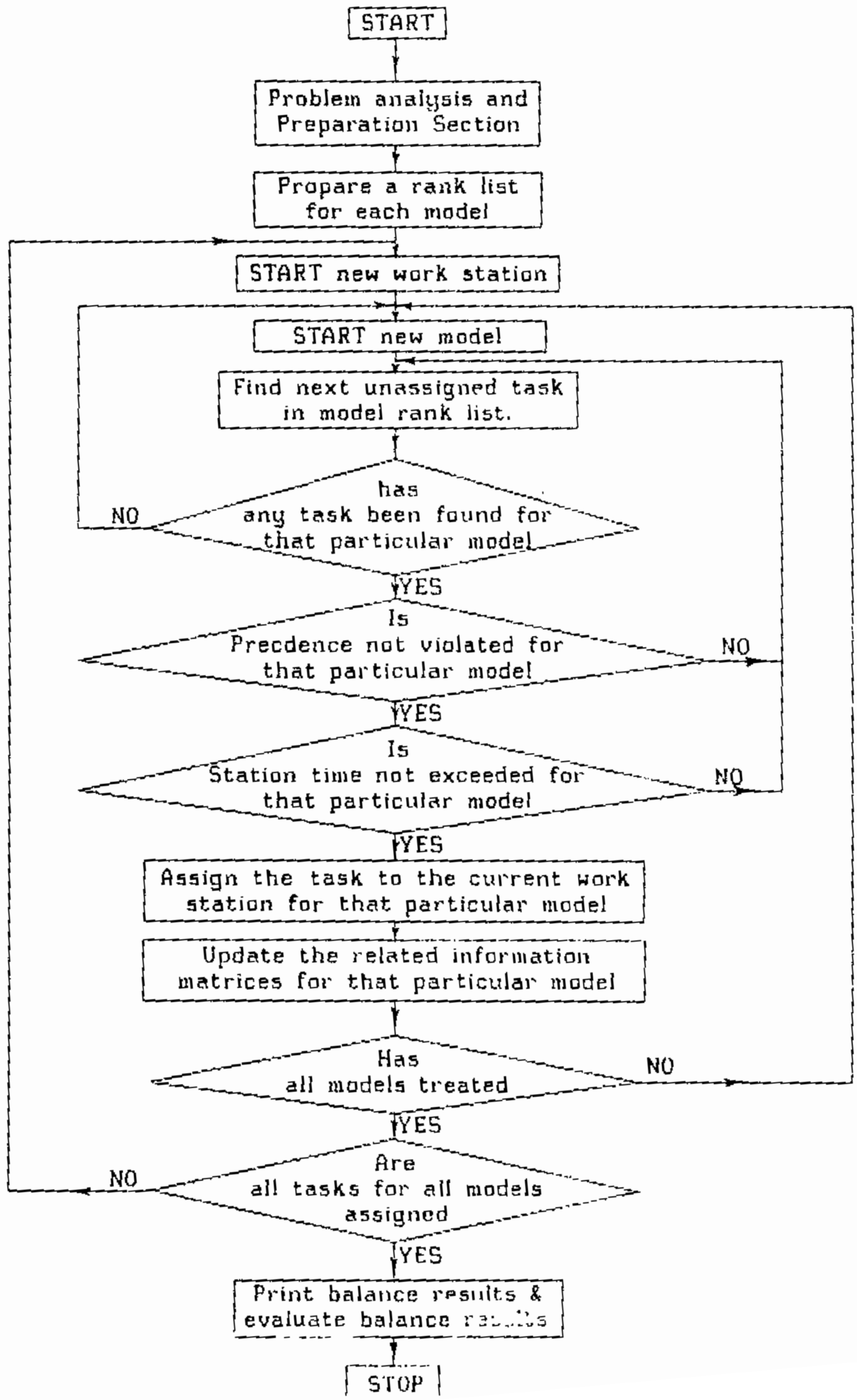
This phase contains the assignment steps as follows :

1- Start new work station
2- Start new model
3- For each model in turn, the current rank is set to one.
4- For that particular model, select the next task in rank order, discount tasks already assigned or failed to be assigned and make the two checks in next step. If no task remains, go to step 7.
5- For the selected task and for the current model check the following
   (i) Is the task free from precedence restrictions point of view
   (ii) Does sufficient time remains in the work station to accommodate the task, i.e.

$$t_i \; (I_j) \leq C - \sum_{q=1}^{n} t_q \; (I_j \, A_k)$$



Program and Files Arra..gement.


START
Problem analysis and Preparation Section
Propare a rank list for each model
START new work station
START new model
Find next unassigned task in model rank list.
has any task been found for that particular model
NO
YES
Is Precdence not violated for that particular model
NO
YES
Is Station time not exceeded for that particular model
NO
YES
Assign the task to the current work station for that particular model
Update the related information matrices for that particular model
Has all models treated
NO
YES
Are all tasks for all models assigned
NO
YES
Print balance results & evaluate balance results
STOP

6- If conditions (i) and (ii) are satisfactory, assign the task to the work station for the current model. Update the relative informations and return to step 3.

7- If further models needed to be treated, repeat steps 3 to 6 for the next model

8- Check if all tasks for all models have been assigned. If yes go to the third phase (Phase III), If no go to step 1.

Where :

$t_i (I_j)$: Duration of task i, where the task is involved in model j

$t_q (I_j A_k)$: Duration of task q, when involved in model j and assigned to work station k.

C : Cycle time or production rate.

n : Number of tasks involved in production for all models

Phase III :

As the work assigned to any work station along the mixed model assembly line is expected to vary from model to model, a new balancing evaluation is used to evaluate the resulted balance. In following the developed criterions used to assess the mixed model balancing.

Station maximum work :

Is the maximum amount of work been assigned to the work station for any of the models served at that station.

Station minimum work :

Is the minimum amount of work been assigned to the work station for any of the models served at that station.

Station mean work :

Is the mean amount of work been assigned to the work station for all models served at that station (performance durations for all models served at the station divided by the number of models).

Station work range :

The difference between the maximum and minimum work durations performed by the station for all models served there.

Station work variety :

The durations summation of the different tasks performed at the station for the different models served there.

Balance delay : (BD%)

The percentage of total idle time along the line for all models been surved to the total production time given for all models. (Without considering the effect of model ratios, i.e. all models having the same weight)

Weighted balance delay : (WBD%)

The weighted percentage of the total idle time along the line for all models to the total production time given for all models. (Without consideration of the model ratios in production schedule. Model weights equal to the production model mix ratios.)

Smoothness index : (SI)

Is a measure of how much the work is evenely distributed among the line work stations (Without consideration of model ratios in the production schedule, i.e. all models having the same weight.)

Weighted smoothness index : (WSI)

Is a measure of how much the work is evenly weighted among the line work stations, considering the model weights effect.

$$BD\% = \frac{\sum_{j=1}^{m} \sum_{k=1}^{k} [\, c - \sum_{i=1}^{n} t_i \, (A_{jk}) \,]}{\sum_{j=1}^{m} \sum_{k=1}^{k} c} \times 100\%$$

$$WBD\% = \frac{\sum_{j=1}^{m} R_j \sum_{k=1}^{k} [\, c - \sum_{i=1}^{n} t_i \, (A_{jk}) \,]}{\sum_{k=1}^{k} c} \times 100\%$$

$$SI = \left[\frac{\sum_{j=1}^{m} \sum_{k=1}^{k} [\, c - \sum_{i=1}^{n} t_i \, (A_{jk}) \,]^2}{m} \right]^{1/2}$$

$$WSI = \left[\sum_{j=1}^{m} R_j \sum_{k=1}^{k} [\, c - \sum_{i=1}^{n} t_i \, (A_{jk}) \,]^2 \right]^{1/2}$$

Where

- $t_i\,(A_{jk})$: Task i duration when both involved in model j and assigned to station k
- C : Production rate or cycle time
- K : Line length or number of work stations
- K : Work station identity number
- m : Number of models in production schedule
- n : Number of tasks involved in mixed-model production
- j : Model identity number
- i : Task identity number.

The computer program :

The developed procedure has been programed in FORTRAN IV. A test case problem is solved and displayed in the following part to show an application of the programed mixed-model balancing rocedure. The program produces the following output.

1- Brief summary for the mixed-model balancing procedure
2- The suitable problem cases for the approach applications
3- All the mixed-model problem data required to define the problem, such as :
   - Summary of the model-task involvements
   - Precedence relationships between the tasks in tabular form
   - Model-task informations (summary for the number of tasks belong to each model, total work durations, and model ratios)
4- Resulting balancing solution in details
5- Summary of the resulting balancing solution.
6- Resulting balancing evalutions, using the evaluation criteria defined in phase three of the procedure.

Conclusions :

The balancing approach presented in this paper offers a solution for the mixed-model balancing problem. The approach considers the model weights in ranking tasks for assignment. Also introduces new methods and ideas for mixed-model balancing evaluations. The developed approach gives very good balancing solutions in cases where the following conditions are existed :

a- Work training is not expensive, as the balance comes up with a variety of work assigned to each work station for the different models.

b- Work content durations is nearly the same for each model and the work content performance is different from model to model.

The approach can be used to balance different products as well as different models of one product. Also the approach can be used as a tool in group technology work where much variety of similar products are produced.

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WORK ELEMENT _ MULTIPLE STATION ASSIGNMENT

* BALANCING EACH WORK STATION FOR INDIVIDUAL MODEL ALLOWING FOR INDIVIDUAL WORK ELEMENTS TO BE ASSIGNED TO MORE THAN ONE WORK STATION FOR THE DIFFERENT INVOLVED MODELS

APPLIED CASES :
****

* MIXED MODEL PRODUCTION

* LITTLE OR NO DIFFERENCE IN MODEL PRODUCTION RATIOS

* WORK TRAINING IS NOT EXPENSIVE

* EQUAL OR DIFFERENT AMOUNT OF WORK CONTENT DURATIONS FOR EACH MODEL
(BUT VERY SUITABLE FOR EQUAL AMOUNT OF WORK CONTENT DURATIONS FOR EACH MODEL)

* SIMILAR OR DIFFERENT WORK CONTENT PERFORMANCE FOR EACH MODEL
(BUT VERY SUITABLE FOR LARGE VARIATION IN WORK CONTENT PERFORMANCE OF MODELS)

MODEL _ ELEMENTS INVOLVEMENT
****

MODEL NUMBER	* INVOLVED WORK ELEMENTS *																		
1000	1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2000	1	2	3	4	6	8	9	10	11	13	14	15	16	17	18	19			
3000	1	2	3	5	6	7	8	9	11	12	14	15	16	17	18	19			
4000	1	2	3	4	6	7	8	9	10	13	14	15	17	18	19				
5000	1	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
6000	1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	
7000	1	2	3	4	5	6	7	8	9	10	11	12	14	17	18				
8000	1	3	4	5	6	8	9	10	11	12	13	14	15	16	17	18	19	20	
9000	1	2	3	4	5	8	9	10	11	13	15	16	17	20					
9999	1	2	3	4	5	6	7	8	9	12	14	15	18	20					

WORK ELEMENT DURATIONS AND MIXED_PRECEDENCE RELATIONSHIPS
****

ELEMENT NUMBER	ELEMENT TIME	IMMEDIATE SUCCEEDING ELEMENTS
1	6.00	2 3 4
2	42.00	5
3	6.00	6
4	9.00	9
5	11.00	7
6	12.00	8 9
7	4.00	10 11
8	17.00	12
9	7.00	13
10	8.00	14
11	4.00	14
12	15.00	15
13	10.00	15 16
14	6.00	17
15	9.00	18
16	5.00	19
17	9.00	20
18	7.00	20
19	6.00	20
20	15.00	NO FOLLOWERS

MIXED MODEL INFORMATIONS
TWENTY ELEMENT - MIXED MODEL BALANCING PROBLEM

NUMBER OF MODELS =10 CYCLE TIME = 42.00 MINUTES

MODEL IDENTITY NUMBER	NUMBER OF ELEMENTS	TOTAL WORK TIME	MODEL QUANTITY	MODEL RATIO
1000	19	166.00	100	0.10
2000	16	163.00	100	0.10
3000	16	166.00	100	0.10
4000	15	158.00	100	0.10
5000	18	157.00	100	0.10
6000	18	160.00	100	0.10
7000	15	163.00	100	0.10
8000	18	162.00	100	0.10
9000	14	158.00	100	0.10
9999	14	166.00	100	0.10

RESULTING MIXED-MODEL BALANCING SOLUTIONS

WORK STATION NUMBER : 1

MODEL NUMBER :	1000	2000	3000	4000	5000	6000	7000	8000	9000	9999
WORK ELEMENT NUMBERS	* INVOLVED WORKING DURATIONS *									
1	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
3	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
4	9.00	9.00	0.00	9.00	0.00	0.00	0.00	9.00	9.00	0.00
6	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	0.00	12.00
8	0.00	0.00	17.00	0.00	17.00	17.00	17.00	0.00	0.00	17.00
9	7.00	7.00	0.00	7.00	0.00	0.00	0.00	7.00	7.00	0.00
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.00
STATION WORKING DURATIONS FOR EACH MODEL	40.00	40.00	41.00	40.00	41.00	41.00	41.00	40.00	38.00	41.00

WORK STATION NUMBER : 2

MODEL NUMBER :	1000	2000	3000	4000	5000	6000	7000	8000	9000	9999
WORK ELEMENT NUMBERS	* INVOLVED WORKING DURATIONS *									
2	0.00	42.00	42.00	42.00	0.00	0.00	42.00	0.00	42.00	42.00
4	0.00	0.00	0.00	0.00	0.00	9.00	0.00	0.00	0.00	0.00
5	11.00	0.00	0.00	0.00	11.00	11.00	0.00	11.00	0.00	0.00
7	4.00	0.00	0.00	0.00	4.00	4.00	0.00	0.00	0.00	0.00
8	17.00	0.00	0.00	0.00	0.00	0.00	0.00	17.00	0.00	0.00
9	0.00	0.00	0.00	0.00	7.00	7.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	8.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.00	0.00	0.00
13	10.00	0.00	0.00	0.00	10.00	10.00	0.00	10.00	0.00	0.00
STATION WORKING DURATIONS FOR EACH MODEL	42.00	42.00	42.00	42.00	40.00	41.00	42.00	42.00	42.00	42.00

WORK STATION NUMBER : 3

MODEL NUMBER :	1000	2000	3000	4000	5000	6000	7000	8000	9000	9999
WORK ELEMENT NUMBERS	* INVOLVED WORKING DURATIONS *									
4	0.00	0.00	0.00	0.00	0.00	0.00	9.00	0.00	0.00	9.00
5	0.00	0.00	11.00	0.00	0.00	0.00	11.00	0.00	11.00	11.00
7	0.00	0.00	4.00	4.00	0.00	0.00	4.00	0.00	0.00	0.00
8	0.00	17.00	0.00	17.00	0.00	0.00	0.00	0.00	17.00	0.00
9	0.00	0.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00	7.00
10	8.00	8.00	0.00	8.00	0.00	8.00	8.00	8.00	8.00	0.00
11	4.00	4.00	4.00	0.00	4.00	4.00	4.00	0.00	4.00	0.00
12	15.00	0.00	15.00	0.00	15.00	15.00	0.00	15.00	0.00	15.00
13	0.00	10.00	0.00	10.00	0.00	0.00	0.00	0.00	0.00	0.00
14	6.00	0.00	0.00	0.00	6.00	6.00	6.00	6.00	0.00	0.00
15	9.00	0.00	0.00	0.00	9.00	9.00	0.00	9.00	0.00	0.00
16	0.00	0.00	0.00	0.00	5.00	0.00	0.00	0.00	0.00	0.00
STATION WORKING DURATIONS	42.00	39.00	41.00	39.00	39.00	42.00	42.00	38.00	40.00	42.00

FOR EACH MODEL

WORK STATION NUMBER : 4

MODEL NUMBER :	1000	2000	3000	4000	5000	6000	7000	8000	9000	9999
WORK ELEMENT NUMBERS	* INVOLVED WORKING DURATIONS *									
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.00
9	0.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	15.00	0.00	0.00	0.00
14	0.00	6.00	6.00	6.00	0.00	0.00	0.00	0.00	0.00	6.00
15	0.00	9.00	9.00	9.00	0.00	0.00	0.00	0.00	9.00	9.00
16	5.00	5.00	5.00	0.00	0.00	5.00	0.00	5.00	5.00	0.00
17	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	0.00
18	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	0.00	7.00
19	6.00	6.00	6.00	6.00	6.00	0.00	0.00	6.00	0.00	0.00
20	15.00	0.00	0.00	0.00	15.00	15.00	0.00	15.00	15.00	15.00
STATION WORKING DURATIONS FOR EACH MODEL	42.00	42.00	42.00	37.00	37.00	36.00	38.00	42.00	38.00	41.00

LINE BALANCE ASSESSMENT:

PERCENTAGE BALANCE DELAY = 3.63

PERCENTAGE WEIGHTED BALANCE DELAY = 3.63

SMOOTHNESS INDEX = 4.57

WEIGHTED SMOOTHNESS INDEX = 4.57

LINE STATIONS WORK ANALYSIS :

STATION NUMBER	MAXIMUM WORK	MINIMUM WORK	MEAN WORK	RANGE WORK	VARIETY WORK
1	41.00	38.00	39.50	3.00	67.00
2	42.00	40.00	41.00	2.00	112.00
3	42.00	38.00	40.00	4.00	105.00
4	42.00	36.00	39.00	6.00	83.00